Progressive Wave Analysis Describing Wave Motions in Radiative Magnetogasdynamics

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An asymptotic analysis has been carried out to study the propagation of weak nonlinear waves in radiative magnetofluids. The progressive wave theory has been used to derive an evolution equation governing the propagation of nonlinear waves. It has been solved for high-frequency domain. The solution indicates the possibility of a wave breaking after a finite time t_c known as the critical time of shock formation. Effects of the thermal radiation, magnetic field, and the coupling of the thermal radiation with the magnetic field have been studied. The decay behavior of a wave with a sawtooth profile headed by a weak shock front and ending with a weak discontinuity has been investigated.

Introduction

THE propagation of weak nonlinear waves has been studied by several researchers using the theory of progressive waves in different material media. Ishii[1] studied the nonlinear wave propagation in a vibrationally relaxing gas. Hunter and Keller[2] presented a method for finding small-amplitude, high-frequency wave solutions of hyperbolic systems of quasilinear partial differential equations. The method applies in any number of space dimensions and reduces to geometrical optics when the equations are linear. A general discussion of small-amplitude nonlinear progressive waves has been presented by Choquet-Bruhat.[3] He considered a shockless solution of hyperbolic partial differential equations that depend on a single phase function. Using the perturbation method devised by Choquet-Bruhat,[3] Germain,[4] Fusco and Engelbreckt,[5] and Fusco[6] gave a general discussion of single phase progressive waves with dissipation and dispersion included. Sharma et al.[7] extended their analysis to magnetogasdynamics.

In this paper we have studied the decay behavior of an N-wave in the form of a sawtooth profile, which consists of a shock front at the right and a gasdynamic weak discontinuity at the left in the high-frequency domain. We have considered the symmetric motion of planar and cylindrical weak nonlinear waves in a plasma with thermal radiation. The plasma is assumed to be an ideal gas with infinite electrical conductivity and permeated by a transverse magnetic field. The gas is taken to be sufficiently hot for effects of thermal radiation to be significant. These effects are treated by the optically thin approximation of the radiative heat transfer equation. We have also investigated how the interaction of the radiative heat transfer with the magnetohydrodynamic (MHD) phenomenon modifies the decay behavior of an N-wave.

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Basic Equations

Assuming the electrical conductivity to be infinite and the direction of the magnetic field orthogonal to the trajectories of gas particles, the basic equations for a one-dimensional planar and cylindrical symmetric motion in radiative magnetogasdynamics can be written in the form[8–10]

$$P_{,t} + up_{,x} + \gamma p\ (u_{,x} + mu/x) + (\gamma - 1)Q = 0 \tag{1}$$

$$\rho_{,t} + u\rho_{,x} + \rho(u_{,x} + mu/x) = 0 \tag{2}$$

$$\rho(u_{,t} + uu_{,x}) + (p_{,x} + h_{,x}) = 0 \tag{3}$$

$$h_{,t} + uh_{,x} + 2h(u_{,x} + mu/x) = 0 \tag{4}$$

where p is the gas pressure, u is the gas velocity, ρ is the density, γ is the heat exponent, and h is the magnetic pressure defined as $h = \mu H^2/2$ with μ as magnetic permeability, and H is the transverse magnetic field. A comma followed by a letter subscript denotes partial differentiation, and the parameter m equals 0 and 1 for planar and cylindrically symmetric motions, respectively. The quantity Q is the rate of energy loss by the gas per unit volume through radiation and is given by $Q = 4k\sigma(T^4 - T_0^4)$, with k as the Planck absorption coefficient. The σ is the Stefan-Boltzmann constant, and T_0 is a constant state temperature. The effect of the thermal radiation is treated by the optically thin approximation of the radiative heat transfer equation.

It is easy to check that the form of Eqs. (1–4) remains unchanged if the time t is normalized by an appropriate characteristic time t^*, the distance x by x_0 ($= a_0 t^*$), the sound speed a by a_0, the density ρ by ρ_0, the gas velocity u by a_0, the gas pressure p and the magnetic pressure h by $\rho_0 a_0{}^2$ and Q by $\rho_0 a_0{}^2/t^*$, where subscript 0 is used to indicate constant reference value. Using the matrix notation, the normalized Eqs. (1–4) can be written in the form

$$U_{i,t} + A_{ij}U_{j,x} + B_i = 0 \tag{5}$$

where $U = U_i)$ is a column vector with four components p,ρ,u,h; the (4×4) matrix $A = (A_{ij})$ and the column vector $B = (B_i)$ can be readoff by inspection of Eqs. (1–4). The system of Eq. (5) is hyperbolic with eigenvalues λ_i ($i = 1,2,3,4$) of the coefficient matrix A and are given by

$$\lambda_1 = u + c, \qquad \lambda_2 = u - c, \qquad \lambda_3 = u = \lambda_4$$

where $c = \sqrt{\{a^2 + b^2\}}$ is the magnetoacoustic speed with $a = \sqrt{(\gamma p/\rho)}$ as the speed of sound and $b = \sqrt{(2h/\rho)}$ the Alfvén speed. The system of Eq. (5) thus admits four families of characteristics; the first two, $dx/dt = \lambda_{1,2}$, represent waves propagating in the forward and backward directions, respectively, and the remaining two form a set of double characteristics representing the particle path or trajectory. Here we are concerned with the situation when the wave front is an outgoing characteristic given by $dx/dt = \lambda_1$. The left and right eigenvectors of the matrix A corresponding to the eigenvalue λ_1 are, respectively,

$$L = (1,0,\rho c,1), \qquad R = (a^2,1,c/\rho,b^2)'$$

where (′) is used to denote transposition.

Small-Amplitude, High-Frequency Progressive Waves

Let us consider the solution of Eq. (5), which may represent a progressive wave. An asymptotic expansion for U may be written as

$$U(x,t) = U^0 + \frac{1}{\omega}\,U^{(1)}(x,t,\xi) + \frac{1}{\omega^2}\,U^2(x,t,\xi) + \ldots \tag{6}$$

where $U^0 = (U)_0$ is a known constant state solution of Eq. (5) such that $B_i(U^0) = 0$. The remaining terms in Eq. (6) characterize a perturbation of progressive wave nature. With τ_a, the attenuation time characterizing the dissipation mechanism and τ_b, the characteristic time scale for the signal, $\omega = \tau_a/\tau_b$ is a large parameter ($\omega \gg 1$). The underlying situation corresponds to the high-frequency wave where the wave frequency is much larger than the characteristic frequency of the medium (see Seymour and Varley[11]). The $\xi(=\omega f)$ is a fast variable with $f(x,t)$ as the phase function to be determined.

In view of expansion of the form given by Eq. (6), A_{ij} and B_i yield the Taylor expansion of the form

$$A_{ij} = A_{ij}^{(0)} + \frac{1}{\omega}\left(\frac{\partial A_{ij}}{\partial U_k}\right)_0 U_k^{(1)} + \cdots \tag{7}$$

$$B_i = \frac{1}{\omega}\left(\frac{\partial B_i}{\partial U_k}\right)_0 U_k^{(1)} + \frac{1}{\omega^2}\left[\left(\frac{\partial B_i}{\partial U_k}\right)_0 U_k^{(2)} + U_j^{(1)}U_k^{(i)}\left(\frac{\partial^2 B_i}{\partial U_j \partial U_k}\right)_0\left(\frac{1}{2}\right)\right] + \cdots \tag{8}$$

By substituting Eqs. (6–8) in Eq. (5), we obtain the following equations:

$$(A_{ij}^{(0)} - \lambda\delta_{ij})U_{j,\xi}^{(1)} = 0 \tag{9}$$

$$(A_{ij}^{(0)} - \lambda\delta_{ij})U_{j,\xi}^{(2)} + (U_{i,t}^{(1)} + A_{ij}^{(0)}U_{j,x}^{(1)})f_{,x}^{-1} + U_k^{(1)}\left(\frac{\partial A_{ij}}{\partial U_k}\right)_0 U_{j,\xi}^{(1)} + f_{,x}^{-1}\left(\frac{\partial B_i}{\partial U_k}\right)_0 U_k^{(1)} = 0 \tag{10}$$

where $\lambda = -f_{,t}/f_{,x}$ and δ_{ij} is the kronecker delta. Equation (9) provides the characteristic equation $\lambda^2(\lambda^2 - c_0^2) = 0$, which yields nonzero eigenvalues $+c_0$ and $-c_0$ of $A_{ij}^{(0)}$. The left and right eigenvectors of $A^{(0)}$ corresponding to an eigenvalue $\lambda = c_0$, computed in the last section are to be used with subscript -0. Equation (9) shows that $U_{j,\xi}^{(1)}$ is collinear to $R_j^{(0)}$ and may be written as

$$U_j^{(1)}(x,t,\xi) = g(x,t,\xi)\; R_j^{(0)} + s_j(x,t) \tag{11}$$

where $g(x,t,\xi)$ is the wave amplitude to be determined, and s_j are integration constants, which are not of the progressive wave nature and can therefore be assumed to be zero. As envisaged earlier, let $f(x,t) = 0$ be the initial wave front trace corresponding to the eigenvalue λ_1 passing through the point (1,0). The phase function $f(x,t)$, determined by $f_{,t} + c_0 f_{,x} = 0$ with the initial condition $f(x,0) = x - 1$, can therefore be expressed as

$$f = x - c_0 t - 1 \tag{12}$$

The gas in the region ahead of the wave front $f(x,t) = 0$ is assumed to be at rest, have a uniform density $\rho_0(=1)$, and a uniform pressure $p_0(=1/\gamma)$ so that $a_0^2 = 1$; consequently, the left and right eigenvectors of $A^{(0)}$ corresponding to an eigenvalue c_0 may be written as

$$L^{(0)} = (1,0,c_0,1), \qquad R^{(0)} = (1,1,c_0,b_0^2)'$$

Eq. (11), in view of the right eigenvector $R^{(0)}$, yields

$$p^{(1)} = \rho^{(1)} = g(x,t,\xi), \qquad u^{(1)} = c_0 g(x,t,\xi),$$
$$h^{(1)} = b_0^2 g(x,t,\xi) \tag{13}$$

Multiplying Eq. (10) by $L^{(0)}$ and using Eqs. (12), (13), and the relation $L_i A_{ij} = \lambda L_j$ and $A_{ij}R_j = \lambda R_i$, we find that the wave amplitude $g(x,t,\xi)$ satisfies the following so-called inviscid Burger-type equation along the wave front $f = 0$:

$$g_{,\tau} + \Gamma c_0 g g_{,\xi} + \Omega g = 0 \tag{14}$$

where $\partial/\partial\tau = \partial/\partial t + c_0\partial/\partial x$ is the ray-derivative taken along the ray

$$\Gamma = \left\{R_i^{(0)}\frac{\partial \lambda}{\partial U_i}\right\}_0 = \frac{3c_0^2 + \gamma - 2}{2c_0^2}$$

and

$$\Omega = \left\{L_i^{(0)}R_j^{(0)}\frac{\partial B_i}{\partial U_j}\right\}_0 / (L_k^{(0)}R_k^{(0)}) = \frac{\Lambda}{c_0^2} + \frac{mc_0}{2x}$$

Here, $\Lambda = 8k(\gamma - 1)\beta^{-1}$ is a measure of thermal radiation with $\beta = \{(\gamma - 1)\sigma_0 T_0^4\}^{-1}$ as the Boltzmann number representing the rate of convective energy flux ($\rho_0 a_0^3$, which is unity in terms of chosen units) to the black body heat flux. Further, it may be noted that in the absence of magnetic field $\Gamma = (\gamma + 1)/2$, $\Gamma \to 3/2$ as $c_0 \to \infty$. The quantity $(\Lambda/c_0^2)^{-1}$ has the dimension of time, which can be taken as the attenuation time characterizing the medium.

Acceleration Waves

The previous analysis may be used to study acceleration waves for Eqs. (1–4). Let us assume that the acceleration front is described by the curve $f(x,t) = 0$. Along such a wave front, U is continuous, but the first- and higher-order derivatives suffer finite jump discontinuities. In the neighborhood of the wave front, the velocity u may be represented by the following expansion:

$$u = \frac{1}{\omega}\,u^{(1)}(x,t,\xi) + 0(1/\omega^2)$$

where $u^{(1)} = (0$, for $\xi < 0$; and $0(\xi)$ for $\xi > 0)$.

The velocity component $u^{(1)}$ of $U^{(1)}$ is given by Eq. (13); thus, we have the following:

$$g(x,t,\xi) = \begin{Bmatrix} 0, & \text{if } \xi < 0 \\ \xi\sigma(x,t) + 0(\xi^2), & \text{if } \xi > 0 \end{Bmatrix} \tag{15}$$

with $\sigma(x,t) = \alpha/c_0$, where $\alpha = [u_{,x}]$ denotes a jump in the velocity gradient across the acceleration wave front. Using

Eq. (15) in Eq. (14) and evaluating at the wave front $f(x,t) = 0$, the strength of discontinuity satisfies the following Bernoulli-type differential equation:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} + \left(\frac{\Lambda}{c_0^2} + \frac{mc_0}{2x}\right)\alpha + \Gamma\alpha^2 = 0 \tag{16}$$

where $\mathrm{d}/\mathrm{d}t$ is an ordinary time derivative along the front $f(x,t) = 0$. The solution of Eq. (16) for $m = 0$ and $m = 1$, respectively, is given by the following two equations:

$$\alpha = \alpha_0 \exp\left(\frac{-\Lambda t}{c_0^2}\right)\left\{1 + \frac{\alpha_0 \Gamma c_0^2}{\Lambda}\left[1 - \exp\left(\frac{-\Lambda t}{c_0^2}\right)\right]\right\}^{-1} \tag{17a}$$

$$\alpha = \frac{a_0 \exp\left(\dfrac{-\Lambda t}{c_0^2}\right) x^{-1/2}}{1 + \alpha_0 \Gamma \left(\dfrac{c_0 \pi}{\Lambda}\right)^{1/2} \exp\left(\dfrac{\Lambda}{c_0^3}\right) \mathrm{erfc}\left(\dfrac{\Lambda}{c_0^3}\right)^{1/2} \left\{\dfrac{\mathrm{erfc}\left(\dfrac{\Lambda x}{c_0^3}\right)^{1/2}}{\mathrm{erfc}\left(\dfrac{\Lambda}{c_0^3}\right)^{1/2}} - 1\right\}} \tag{17b}$$

where erfc is an error function defined as

$$\mathrm{erfc}(\xi) = \frac{2}{\sqrt{\pi}} \int_{\xi}^{\infty} e^{-t^2}\,\mathrm{d}t, \qquad x = 1 + c_0 t$$

and α_0 ($\neq 0$) is the value of α at time $t = 0$. In a planar motion, a compressive wave α_0 (<0) steepens into a shock wave ($|\alpha| \to \infty$ as $t \to t_s$) only when $|\alpha_0|$ exceeds a critical value $\alpha_1 = \Lambda/\Gamma c_0^2$ where t_s is the shock formation time given by

$$t_s = \frac{c_0^2}{\Lambda} \log\left(1 + \frac{\alpha_1}{\alpha_0}\right)^{-1}$$

An expansion wave ($\alpha_0 > 0$) decays with time and flattens out ultimately. The coupling effect of thermal radiation and magnetic field is to accelerate the process of decaying of expansion waves for all physically realistic values of γ in the range of $1 < \gamma < 2$.

For $|\alpha_0| < \alpha_1$, all compressive waves also decay and flatten out ultimately; whereas for $|\alpha_0| = \alpha_1$, the wave takes a stable form. It is important to note that the effect of radiation increases the critical value α_1, whereas the effect of magnetic field decreases the value of α_1. Thus, the radiation may be regarded as an agent resisting the shock formation in the sense that it enhances the shock formation time t_s. In contrast to the corresponding result obtained by Sharma et al.[7] in the MHD case, we find that in radiative magnetogasdynamics only those compression waves can grow into shock waves for which the magnitude of the initial jump discontinuity associated with the wave exceeds a critical value. This result is also true in the absence of the magnetic field ($c_0 = 1$), as is evident from Eq. (17). Similarly, in a cylindrical ($m = 1$) motion, all expansion waves decay, whereas not all compression waves can grow into shock waves. In fact, a shock wave will be formed if $|\alpha_0| > \alpha_2$, where

$$\alpha_2 = \Gamma^{-1}\left(\frac{\Lambda}{c_0 \pi}\right)^{1/2} \exp\left(\frac{-\Lambda}{c_0^3}\right) \Big/ \mathrm{erfc}\left(\frac{\Lambda}{c_0^3}\right)^{1/2}$$

and the shock formation distance x_s^c can be found from the equation

$$\mathrm{erfc}\left\{\frac{\Lambda x_s^c}{c_0^3}\right\}^{1/2} = \left(1 + \frac{\alpha_2}{\alpha_0}\right) \mathrm{erfc}\left(\frac{A}{c_0^3}\right)^{1/2}$$

For $|\alpha_0| < \alpha_2$, all compression waves decay, whereas for $|\alpha_0| = \alpha_2$, α depends on x as

$$\alpha = \alpha_0 \exp\left(\frac{-\Lambda t}{c_0^2}\right) x^{-1/2} \frac{\mathrm{erfc}\left(\dfrac{\Lambda}{c_0^3}\right)^{1/2}}{\mathrm{erfc}\left(\dfrac{\Lambda t}{c_0^3}\right)^{1/2}}$$

and approaches to α_0 for large x. It is noticeable that the critical value α_2 of the initial jump discontinuity in cylindrical motion is larger than α_1 in the plane case. Conclusions drawn here are in full agreement with the results obtained by Sharma.[12]

Weak Shock

The foregoing analysis shows that in radiative magnetogasdynamics only those compression waves can grow into shock waves for which the magnitude of the initial jump discontinuity associated with the wave exceeds a critical value; the state of motion in front and behind the shock associated with Eqs. (1–4) satisfies the following shock conditions:

$$[\rho(u - V_S)] = 0, \quad [\rho u(u - V_S) + p + h] = 0, \quad [h(u - V_S)^2] = 0 \tag{18a}$$

$$[(u - V_S)\{p/(\gamma - 1) + h + \rho u^2/2\} + u(p + h)] = 0 \tag{18b}$$

where V_S is the shock velocity and $[\rho] = \rho_1 - \rho_0$. Here ρ_1 and ρ_0 are values of ρ evaluated just at the rear and just ahead of the shock respectively. Since there is no absorption or emission within the shock discontinuity, we have $Q_1 = Q_0$. The shock conditions of Eq. (18) can be expressed in terms of the shock strength $[\rho] = \delta$ as follows:

$$\rho_1 = 1 + \delta, \qquad u_1 = \delta V_S (1 + \delta)^{-1}, \qquad h_1 = h_0 (1 + \delta)^2 \tag{19a}$$

$$p_1 = p_0 + \delta V_s^2 (1 + \delta)^{-1} - h_0 \delta(\delta + 2) \tag{19b}$$

where δ and V_s are related to each other by

$$V_s^2 = 2(1 + \delta)[1 + b_0^2\{1 + (2 - \gamma)\delta/2\}]\{2 - (\gamma - 1)\delta\}^{-1} \tag{20}$$

If the shock is assumed to be a weak shock ($\delta \ll 1$), to a first approximation, Eq. (20) gives

$$V_S = c_0(1 + \Gamma\delta/2) \tag{21}$$

where Γ is the same as in Eq. (14). Subsequently Eqs. (19) yield to the first approximation that

$$\rho_1 = 1 + \delta, \quad u_1 = c_0\delta, \quad p_1 = \delta + (1/\gamma), \quad h_1 = h_0(1 + 2\delta) \tag{22a}$$

Decay of Half-N Wave (Sawtooth Profile)

At a large distance from the source, the velocity field of a sufficiently weak shock may be assumed in the form of a sawtooth profile, which is initially of length L_0 such that the left end of the profile located at x_0 travels with speed c_0 of the undisturbed fluid, and the shock on the right end at x_{s_0} moves at the faster rate (see Fig. 1); the physical situation permits an application of the weak shock relations of Eqs. (21) and (22).

Now suppressing the subscript 1 notation, let us denote by u and c the state at the rear side of the shock which at any time t is located at $x_s(t) = 1 + c_0 t + L(t)$, where $L(t)$ is the length of the sawtooth profile at any time t. The shock velocity V_S is given by

$$V_S = \mathrm{d}x_S/\mathrm{d}t = c_0 + \mathrm{d}L/\mathrm{d}t \tag{22b}$$

Eqs. (21) and (22a) give

$$V_S = c_0 + \Gamma u/2 \tag{23}$$

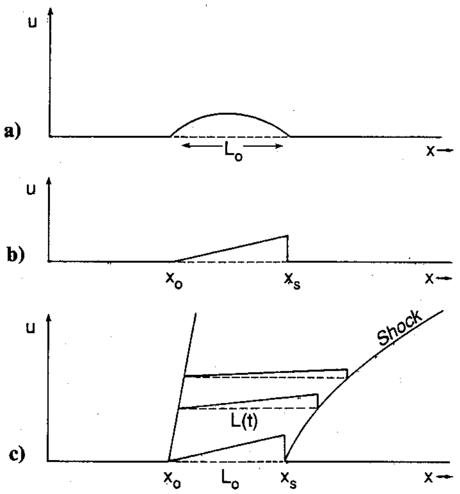

Fig. 1 Formation and decay of a sawtooth profile.

The physical situation envisaged here is as follows: let the initial velocity profile $u(x)$ be zero outside a certain interval of length L_0 and $u(x) > 0$ in this interval (see Fig. 1a); then as time goes on, the compressive part of the profile steepens into a shock, which propagates with the velocity V_S into the constant region $u = 0$. Thus the asymptotic form of the velocity profile is in the shape of a sawtooth (also known as half-N wave) as shown in Fig. 1b. The jump of gas velocity across the shock, located at x_S, is equal to u (i.e., the value at the rear side of the shock) because the medium ahead of it is at rest. Thus the jump in the velocity across the shock located at x_S at any time t is given by

$$u = \alpha L(t) \tag{24}$$

where α is the slope of the expansive part of the profile at time t and is given by Eq. (17). Using Eq. (24) in Eq. (23) and combining the resulting equation with Eq. (22), we find that

$$\mathrm{d}L/\mathrm{d}t = \Gamma\alpha L/2 \tag{25}$$

Evaluating Eqs. (23) and (24) at time $t = 0$, we get

$$\alpha_0 = 2(V_{S_0} - c_0)/L_0\Gamma \tag{26}$$

where α_0, L_0 and V_{S_0} are the values of α, L, and V_S, respectively, at $t = 0$. Eqs. (25) and (17) yield

$$L = L_0\left\{1 + \frac{\Gamma\alpha_0 c_0^2}{\Lambda}\left(-\mathrm{e}^{-\Lambda t/c_0^2} + 1\right)\right\}^{1/2} \quad (\text{for } m = 0) \tag{27a}$$

$$L = L_0\left\{1 + \Gamma\alpha_0\left(\frac{c_0\pi}{L}\right)^{1/2}\exp\left(\frac{\Lambda}{c_0^3}\right)\operatorname{erfc}\left(\frac{\Lambda}{c_0^3}\right)^{1/2} \times \left[1 - \frac{\operatorname{erfc}\left(\frac{\Lambda x}{c_0^3}\right)^{1/2}}{\operatorname{erfc}\left(\frac{\Lambda}{c_0^3}\right)^{1/2}}\right]\right\}^{1/2} \quad (\text{for } m = 1) \tag{27b}$$

Using Eqs. (27) and (17) in Eq. (24), the velocity field in a sawtooth profile will be given by

$$u = L_0\alpha_0 \exp\left(\frac{-\Lambda t}{c_0^2}\right)\left\{1 + \frac{\Gamma\alpha_0 c_0^2}{\Lambda}\left[1 - \exp\left(\frac{-\Lambda t}{c_0^2}\right)\right]\right\}^{-1/2} \quad (\text{for } m = 0) \tag{28a}$$

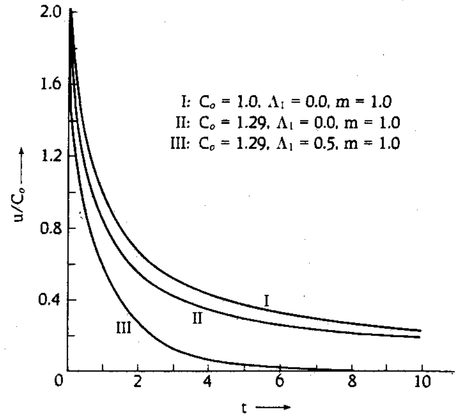

Fig. 2 Velocity decay in a sawtooth profile of cylindrical waves under the effect of radiation and magnetic field.

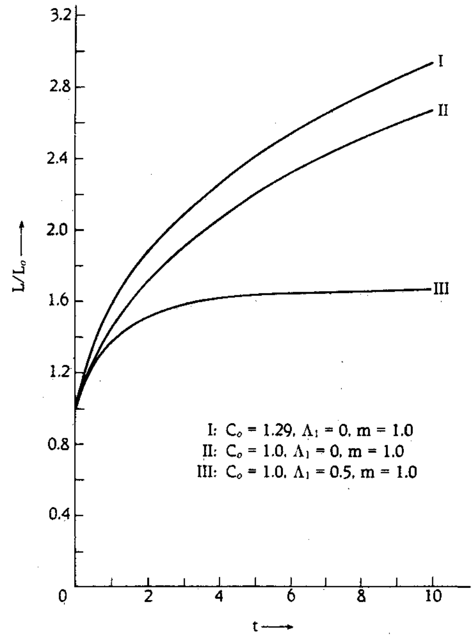

Fig. 3 Coupling effect of the radiation and the magnetic field on the length of a sawtooth profile for cylindrical waves.

$$u = \frac{L_0\alpha_0 \exp\left(\frac{-\Lambda t}{c_0^2}\right)x^{-1/2}}{\left[1 + \Gamma\alpha_0\left(\frac{c_0\pi}{L}\right)^{1/2}\exp\left(\frac{\Lambda}{c_0^3}\right)\operatorname{erfc}\left(\frac{\Lambda}{c_0^3}\right)^{1/2}\left\{1 - \frac{\operatorname{erfc}\left(\frac{\Lambda x}{c_0^3}\right)^{1/2}}{\operatorname{erfc}\left(\frac{\Lambda}{c_0^3}\right)^{1/2}}\right\}\right]^{1/2}} \quad (\text{for } m = 1) \tag{28b}$$

The curves drawn in Fig. 2 show the decay behavior of an N-wave with a sawtooth profile under the effect of a magnetic field and its coupling with the radiation. The curve II shows that the presence of a magnetic field enhances the decay of the sawtooth profile. The curve III shows that under the combined effect of the magnetic and radiation fields, the decaying is further hastened.

Figure 3 shows how the length of the sawtooth is affected by introducing radiation and a magnetic field in the flowfield. The curve I shows that the magnetic field increases the length of the sawtooth profile; whereas the radiation decreases the length. This is contrary to their effects on the velocity profile as is shown in Fig. 2. When their effects are counted together,

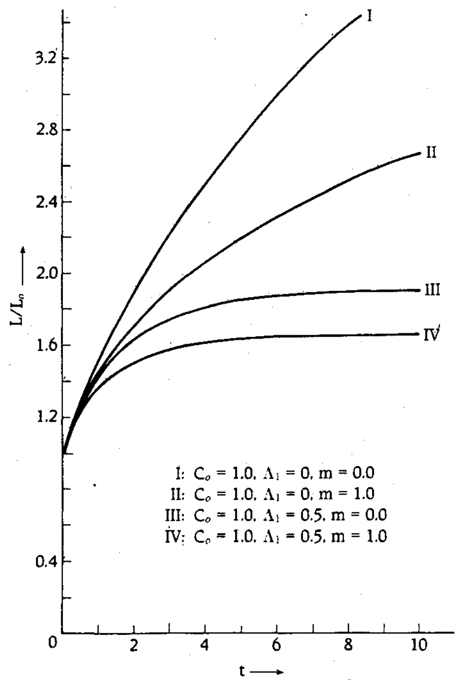

Fig. 4 Effect of radiation on the length L/L_0 of a sawtooth profile for planar and cylindrical waves.

they adjust their positive and negative effects in not decreasing the length of the sawtooth profile. It is evident from curves I, II, and III that there is a competitive coupling effect of the magnetic field and the radiating field in the sense that one overcomes the effect of the other.

Figure 4 shows the effect of the wave geometry on the length of the sawtooth profile and compares the degree of influence of radiation in the case of planar and cylindrical wave fronts. Comparison of curves I and II shows that the sawtooth profile decays faster in the case of planar symmetry than in the case of cylindrical symmetry. Comparison of either curves I and III or curves II and IV shows that the radiation has a stabilizing effect on the sawtooth profile.

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